FORM 3SBP

The College Board
Advanced Placement Examination

CALCULUS AB
SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

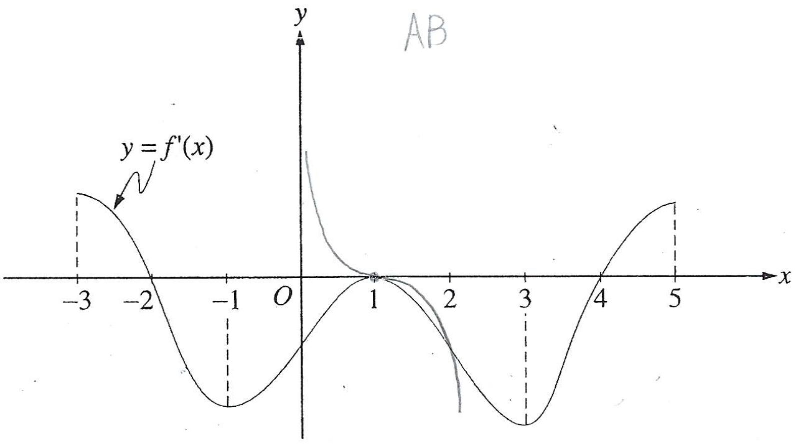
Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of the test booklet.



Note: This is the graph of the derivative of f, not the graph of f.

1. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

   (a) For what values of x does f have a relative maximum? Why?

   (b) For what values of x does f have a relative minimum? Why?

   (c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

   (d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

   Note: The axes for this graph are provided in the pink booklet only.

When you do f'(x) ______

do you put values in

orig. zeros of f'(x) and max/min of f(x)

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2. Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$. AB

(a) Find the area of R.

(b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

(c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

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3. The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

(a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.

(b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure. ws can gsv wc $\frac{S(6) - S(5)}{?-?}$

1983~1993

(c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

(d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

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amount of coca cola consumed between 1985 and 1987

4. This problem deals with functions defined by $f(x) = x + b \sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$.

(a) Sketch the graphs of two of these functions, $y = x + \sin x$ and $y = x + 3 \sin x$, as indicated below.

Note: The axes for these two graphs are provided in the pink test booklet only.

(b) Find the x-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b \sin x$. Slope same and set equal) Answers 2

(c) Are the points of tangency described in part (b) relative maximum points of f? Why?

(d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

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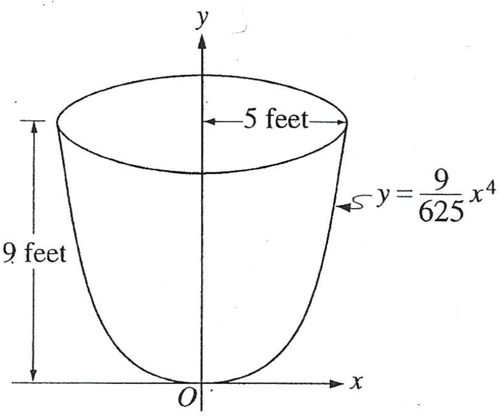

5. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at he constant rate of 8 cubic feet per minute.

(a) Find the volume of the tank. Indicate units of measure.

(b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?

(c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.

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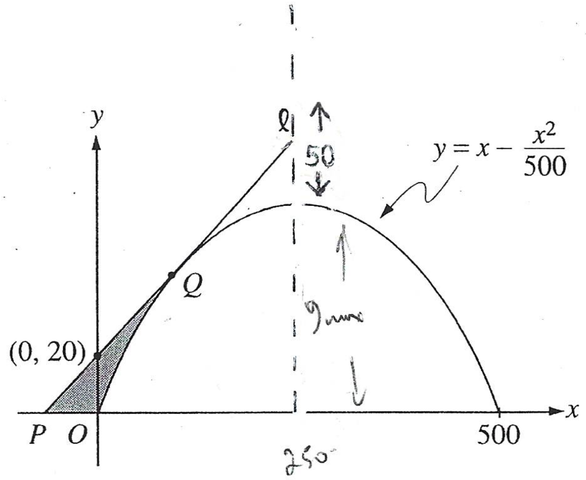

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6. Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.

(a) Find the x-coordinate of point Q.

(b) Write an equation for line ℓ.

(c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

END OF EXAMINATION